Accommodating and extending various models for special effects within the Generalized Partially Confirmatory Factor Analysis (GPCFA) Framework

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Introduction

- Special effects including **Method effects** and **testlet effects** are common issues in educational and psychological measurement.
- Existing models have lot limitations for special effects: Bifactor model, MTMM (multiple traits multiple methods) models, testlet models

Objectives

- Accommodating the GPCFA framework to special effects with added benefits:
  - Partially confirmatory knowledge
  - Local dependence
  - Mixed-type formats
  - Missingness
- Link to various Bifactor, MTMM and testlet effect models:
  - Standard Bifactor
  - CTCU (correlated trait correlated uniqueness)
  - CTUM (correlated trait uncorrelated method)
  - CTCM (correlated trait correlated method), CTC(M-1) (correlated trait correlated method model with one method less)
  - The general testlet model
  - 2PNO testlet model
  - Rasch testlet model
- Provide a subroutine to compute the equivalent effect size.

Theoretical Framework

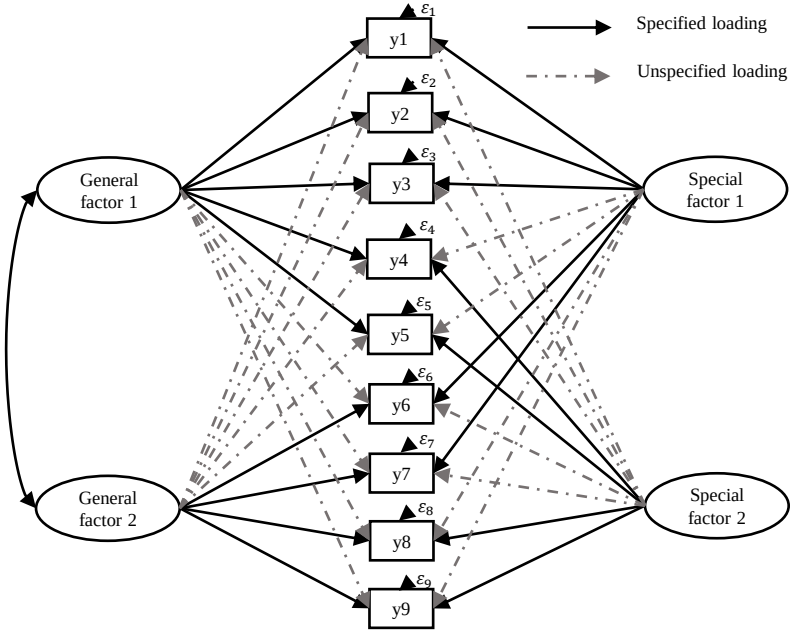
GPCFA for special effects:

$$\mathbf{Y} = \boldsymbol{\mu} + \boldsymbol{\Lambda}_g \mathbf{F}_g + \boldsymbol{\Lambda}_s \mathbf{F}_s + \mathbf{E}$$

- $\mathbf{Y}$ is the observed variables
- $\boldsymbol{\mu}$ represents the $J \times 1$ intercept vector
- matrix $\boldsymbol{\Lambda}_g$ ($\boldsymbol{\Lambda}_s$) represents $J \times K_G$ ($J \times K_S$) general (special) loading matrix
- $\mathbf{F}_g$ ($\mathbf{F}_s$) represents K_G (K_S) factors with the $K_G \times K_G$ ($K_S \times K_S$) factorial covariance matrix $\boldsymbol{\Phi}$
- $\mathbf{E}$ represents the $J \times 1$ residuals with the $J \times J$ residual covariance matrix $\boldsymbol{\Psi}$

Effect Size:

$$\frac{\text{eigenvalue of special factors}}{\text{the number of indicators}}$$



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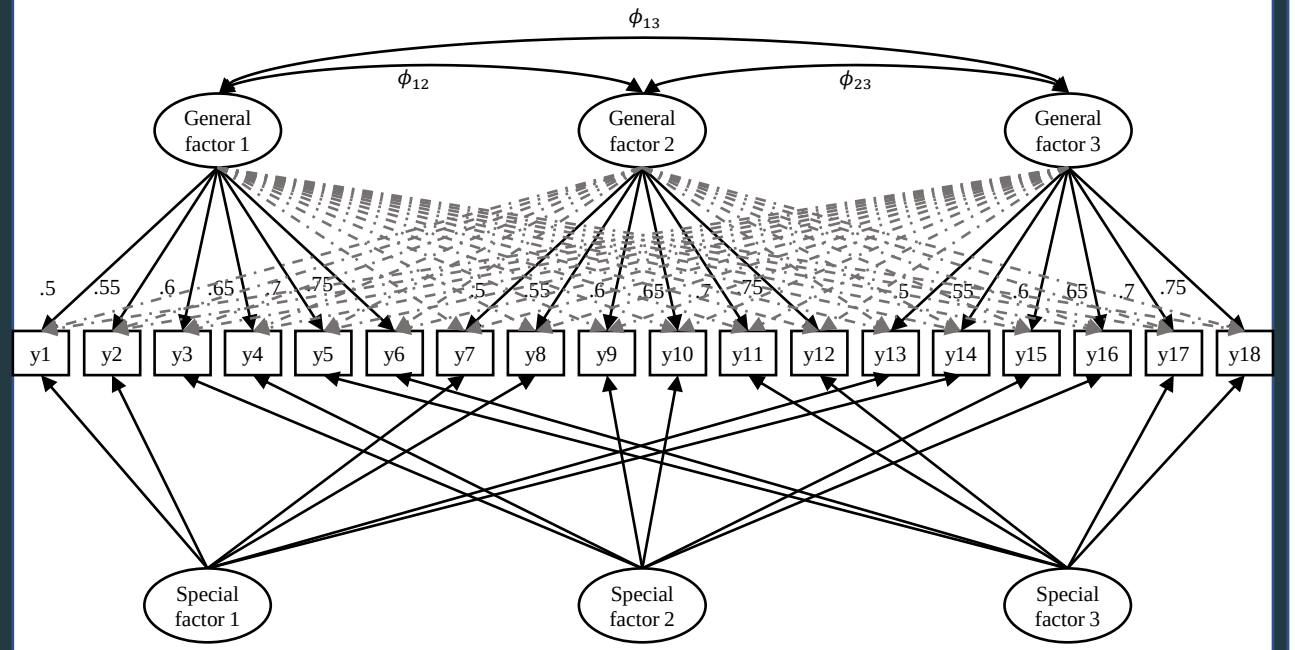
Simulation Study 1 Continuous variables

Data generation

- No. Replication = 200
- Sample size = 1000
- Interference $\psi_{13} = \psi_{31} = \psi_{23} = \psi_{32} = \psi_{46} = \psi_{64} = \psi_{56} = \psi_{65}$ =0.2



Simulation conditions

- Effect Size = 0.1 / 0.2
- Factorial correlation = .3 / .6

Results

Par	True	ϕ = 0.3 BIAS	RMSE	SE	SIG%	ϕ = 0.6 BIAS	RMSE	SE	SIG%
				D = 0.1					
λ_{g1}	0.5	0.002	0.036	0.086	0.998	−0.013	0.044	0.139	1.000
λ_{g2}	0.55	0.001	0.032	0.087	1.000	−0.019	0.044	0.140	1.000
λ_{g3}	0.6	−0.002	0.042	0.086	1.000	−0.019	0.055	0.135	1.000
λ_{g4}	0.65	0.005	0.049	0.089	1.000	−0.021	0.060	0.136	1.000
λ_{g5}	0.7	−0.018	0.038	0.082	1.000	−0.054	0.064	0.127	1.000
λ_{g6}	0.75	−0.003	0.025	0.085	1.000	−0.042	0.052	0.131	1.000
λ_{s1}	0.316	−0.022	0.101	0.122	0.650	−0.025	0.102	0.122	0.632
D_1	0.1	0.003	0.012	0.025	1.000	0.003	0.012	0.025	1.000
D_2	0.1	0.013	0.018	0.024	1.000	0.013	0.018	0.024	1.000
D_3	0.1	0.030	0.033	0.027	1.000	0.026	0.030	0.027	1.000
$\lambda_{g/s0}$	0	0.010	0.031	0.097	0.000	0.025	0.042	0.131	0.000
ψ_{13}	0.2	−0.036	0.047	0.078	0.620	−0.037	0.047	0.076	0.625
ψ_{23}	0.2	−0.038	0.048	0.082	0.450	−0.040	0.049	0.081	0.425
ψ_{46}	0.2	−0.052	0.058	0.101	0.025	−0.035	0.041	0.096	0.190
ψ_{56}	0.2	−0.139	0.139	0.068	0.000	−0.132	0.133	0.072	0.000
$\phi_{kk'}$	0.3/0.6	−0.040	0.054	0.168	0.082	−0.088	0.094	0.196	0.968
				D = 0.2					
λ_{g1}	0.5	−0.009	0.061	0.127	0.938	−0.059	0.102	0.198	0.707
λ_{g2}	0.55	−0.008	0.060	0.135	0.942	−0.063	0.106	0.211	0.740
λ_{g3}	0.6	−0.037	0.085	0.119	0.938	−0.078	0.133	0.192	0.805
λ_{g4}	0.65	0.015	0.145	0.118	0.947	−0.044	0.191	0.181	0.812
λ_{g5}	0.7	−0.034	0.079	0.119	0.958	−0.109	0.140	0.191	0.825
λ_{g6}	0.75	0.009	0.068	0.121	0.958	−0.068	0.126	0.191	0.838
λ_{s1}	0.447	−0.064	0.186	0.118	0.704	−0.081	0.194	0.126	0.596
D_1	0.2	−0.042	0.053	0.042	1.000	−0.047	0.057	0.043	1.000
D_2	0.2	0.061	0.064	0.033	1.000	0.068	0.071	0.034	1.000
D_3	0.2	−0.011	0.027	0.032	1.000	−0.039	0.046	0.035	1.000
$\lambda_{g/s0}$	0	0.007	0.060	0.131	0.000	0.042	0.089	0.194	0.000
ψ_{13}	0.2	−0.029	0.046	0.065	0.830	−0.018	0.042	0.071	0.830
ψ_{23}	0.2	−0.030	0.045	0.069	0.760	−0.018	0.042	0.075	0.735
ψ_{46}	0.2	−0.152	0.153	0.050	0.000	−0.133	0.135	0.065	0.000
ψ_{56}	0.2	−0.195	0.195	0.014	0.000	−0.191	0.191	0.022	0.000
$\phi_{kk'}$	0.3/0.6	−0.025	0.079	0.210	0.158	−0.166	0.203	0.298	0.348

Note. λ_{g1}–λ_{g6} averaged across all general factors; λ_{s1} averaged across all special factors; $\lambda_{g/s0}$ averaged across all zero loading estimates; *D*: effect size; For $\phi_{kk'}$, k and k' = 1 to 3 and $k \neq k'$; RMSE: root mean square error; SE: standard error; SIG%: percent of estimates differed from zero significantly ($\alpha = .05$).

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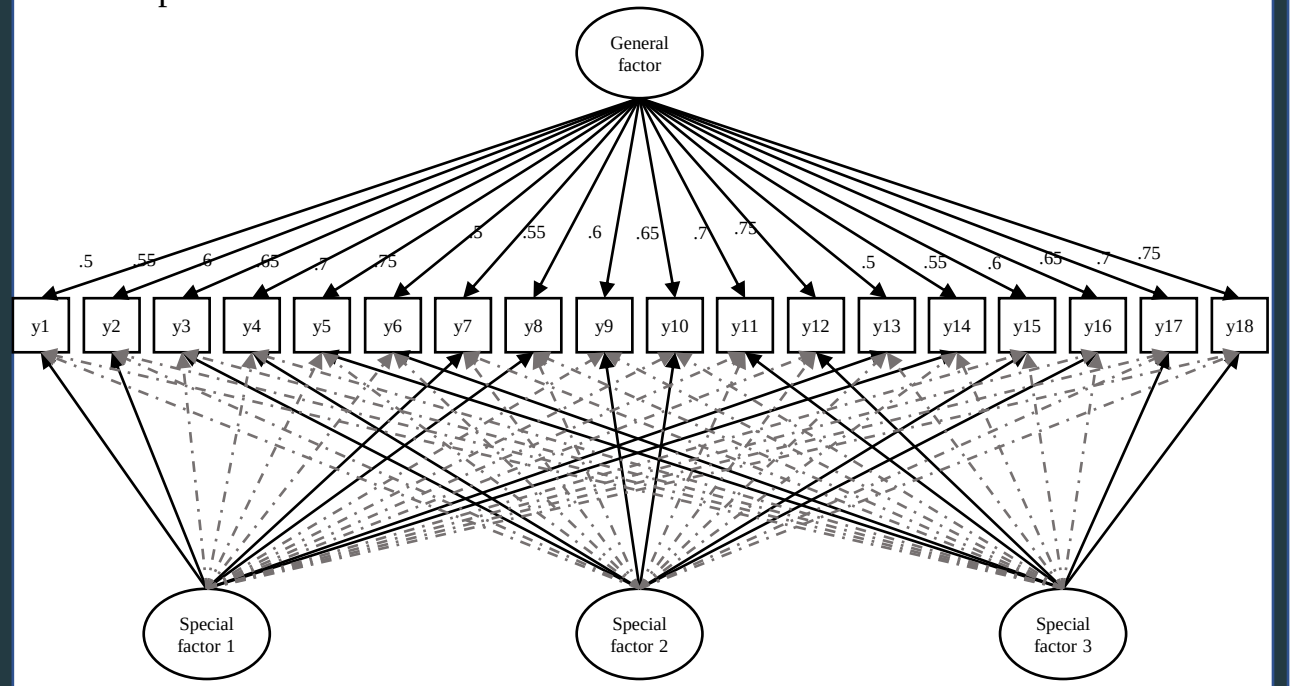
Simulation Study 2 Categorical variables

Data generation

- No. Replication = 200
- Sample size = 1000



Simulation conditions

- Effect Size = 0.1 / 0.2
- Number of categories = 2 / 4

Results

		M = 2				M = 4			
Par	True	BIAS	RMSE	SE	SIG%	BIAS	RMSE	SE	SIG%
				D = 0.1					
λ_{g1}	0.5	−0.004	0.037	0.064	1.000	0.001	0.030	0.060	1.000
λ_{g2}	0.55	−0.002	0.035	0.063	1.000	−0.005	0.029	0.059	1.000
λ_{g3}	0.6	0.000	0.032	0.064	1.000	0.002	0.026	0.061	1.000
λ_{g4}	0.65	−0.005	0.030	0.064	1.000	−0.004	0.024	0.061	1.000
λ_{g5}	0.7	0.004	0.029	0.064	1.000	0.002	0.023	0.063	1.000
λ_{g6}	0.75	0.001	0.027	0.064	1.000	−0.001	0.021	0.062	1.000
λ_{s1}	0.316	−0.053	0.075	0.167	0.101	−0.038	0.058	0.148	0.243
D_1	0.1	−0.019	0.023	0.030	1.000	−0.015	0.019	0.030	1.000
D_2	0.1	−0.016	0.020	0.034	1.000	−0.015	0.019	0.037	1.000
D_3	0.1	−0.017	0.022	0.042	1.000	−0.015	0.019	0.044	1.000
$\lambda_{g/s0}$	0	0.019	0.037	0.113	0.000	0.018	0.032	0.099	0.000
				D = 0.2					
λ_{g1}	0.5	0.003	0.040	0.082	1.000	−0.003	0.034	0.078	1.000
λ_{g2}	0.55	−0.005	0.039	0.080	1.000	−0.012	0.035	0.077	1.000
λ_{g3}	0.6	−0.007	0.037	0.083	1.000	−0.006	0.031	0.080	1.000
λ_{g4}	0.65	−0.014	0.038	0.083	1.000	−0.013	0.031	0.081	1.000
λ_{g5}	0.7	0.021	0.041	0.088	1.000	0.012	0.032	0.086	1.000
λ_{g6}	0.75	0.014	0.037	0.088	1.000	0.004	0.027	0.086	1.000
λ_{s1}	0.447	−0.048	0.079	0.165	0.567	−0.028	0.058	0.145	0.775
D_1	0.2	−0.039	0.045	0.054	1.000	−0.029	0.035	0.054	1.000
D_2	0.2	−0.036	0.043	0.061	1.000	−0.030	0.036	0.065	1.000
D_3	0.2	−0.059	0.066	0.078	1.000	−0.049	0.055	0.079	1.000
$\lambda_{g/s0}$	0	0.010	0.034	0.106	0.001	0.013	0.029	0.094	0.000

Note. λ_{gj} = average of three parts of general factor loadings; λ_{s1} averaged across all special factors; *D*: effect size; $\lambda_{g/s0}$ averaged across all zero loading estimates; *M*: number of categories; RMSE: root mean square error; SE: standard error; SIG %: percent of estimates differed from zero significantly ($\alpha = .05$).

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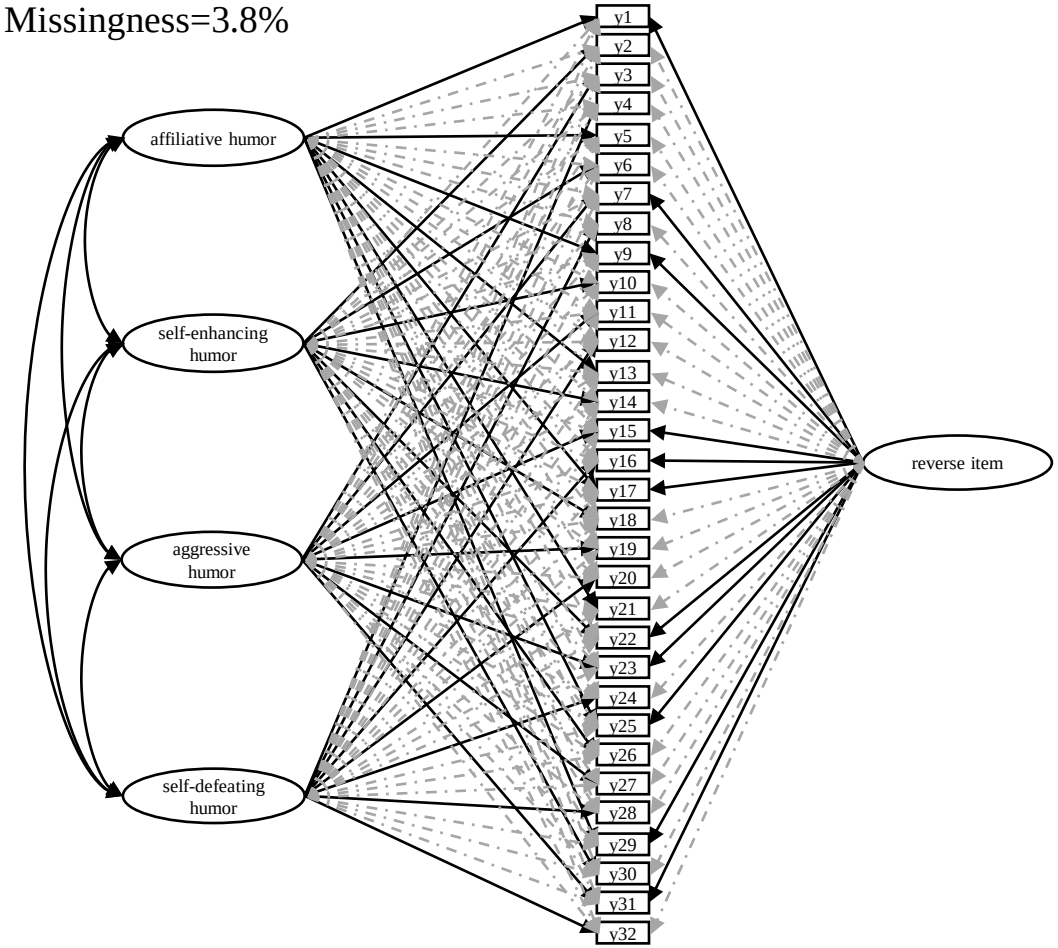
Empirical Study 3 Humor Styles Questionnaire

- N=1070
- Missingness=3.8%



Results

	Local Independent					Local Dependent						
Item	F1	F2	F3	F4	R	F1	F2	F3	F4	R	LD	effect
1	0.668					0.657				0.146*	Ψ24,4	0.114
2		0.636					0.618				Ψ29,5	-0.066
3			0.536	0.094				0.550			Ψ30,6	0.071
4				0.626					0.618		Ψ20,8	0.149
5	0.653				-0.136*	0.612					Ψ18,10	0.247
6	0.215	0.379				0.187	0.382				Ψ25,13	0.141
7	-0.169		0.601			-0.139		0.550		0.177*	Ψ21,17	0.131
8				0.784					0.735		Ψ29,23	0.098
9	0.476			0.092		0.447				0.192	Ψ28,27	-0.076
10	-0.118*	0.816					0.735					
11	-0.122		0.550					0.542				
12				0.640					0.638			
13	0.620				0.269*	0.613						
14		0.657					0.677					
15			0.680					0.596		0.309		
16			0.104	0.556					0.596	0.285		
17	0.773					0.760				0.166		
18	-0.167	0.827					0.732					
19	0.133	0.125	0.444			0.195		0.464				
20				0.782					0.719			
21	0.696		-0.135			0.702						
22		0.375	0.081				0.419			0.233		
23			0.497		0.149*			0.463		0.252*		
24	-0.168			0.478		-0.111*			0.461	-0.106*		
25	0.703				0.366*	0.646				0.174		
26		0.686					0.692					
27			0.548					0.552				
28		0.178	0.154	0.240		0.108*	0.136	0.163	0.241			
29	0.595		0.149	-0.192		0.532			-0.160	0.232		
30		0.454		-0.123			0.447					
31			0.692		0.110*			0.597		0.342		
32				0.660					0.655			
ES					0.039					0.060		

Note. F1 = affiliative humor; F2 = self-enhancing humor; F3 = aggressive humor; F4 = self-defeating humor; R = reverse item; LD = local dependence (only significant terms were presented); ES: effect size; underscored in general factors (F1-F4) are cross-loadings; underscored in special factor (R) are specified items; only significant loadings at general factors and loading absolute value above 0.1 at special factors are presented.

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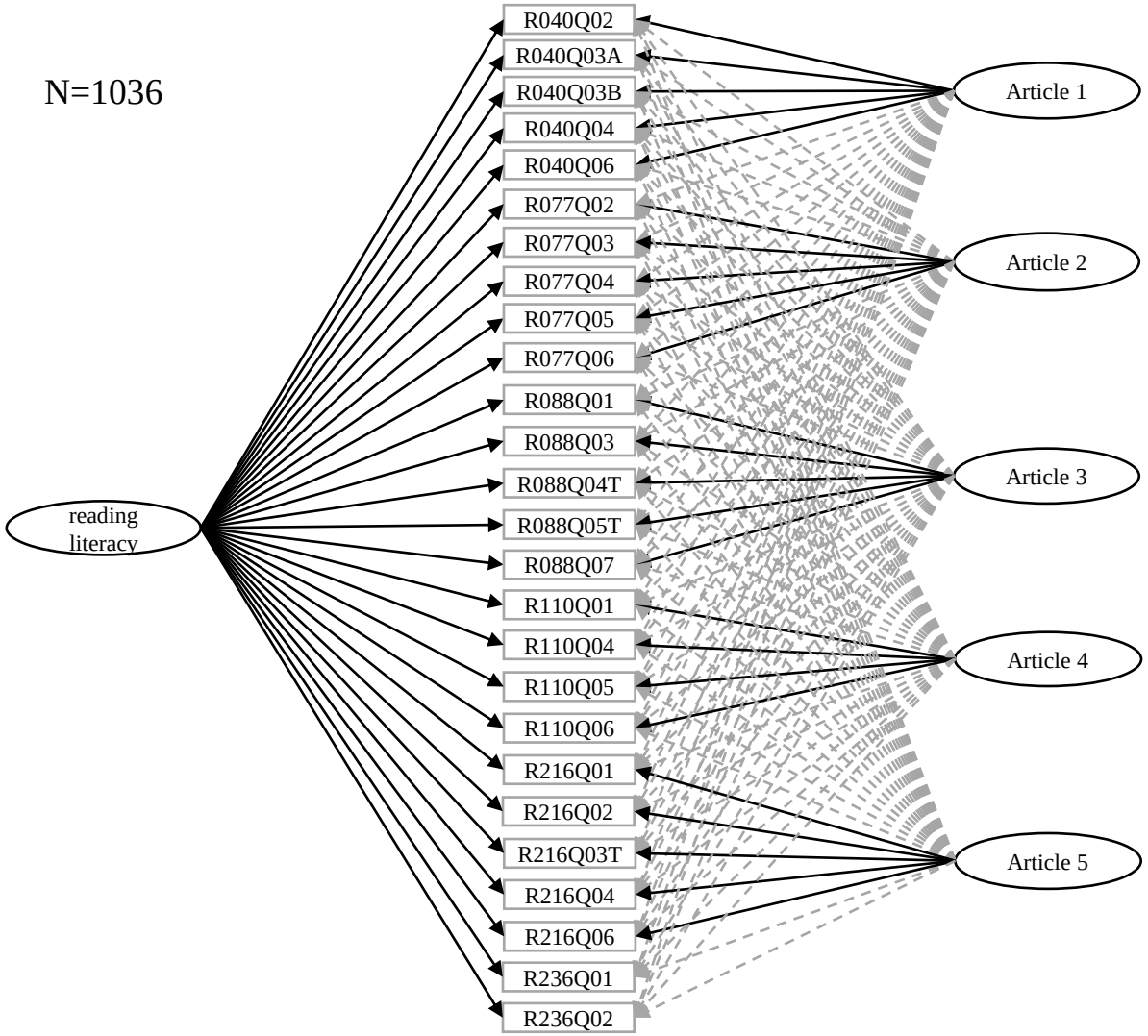
Empirical Study 4 PISA Reading Assessment (UK 2000)

- N=1036



Results

Item	Item code	G1	A1	A2	A3	A4	A5
1	R040Q02	0.543	0.339				
2	R040Q03A	0.625	0.406		0.118*		
3	R040Q03B	0.752	0.448				
4	R040Q04	0.608	0.324				
5	R040Q06	0.555	0.287				
6	R077Q02	0.656		0.141*			
7	R077Q03	0.710		0.219			
8	R077Q04	0.560					
9	R077Q05	0.540		0.393			
10	R077Q06	0.515		0.265			
11	R088Q01	0.684			0.128*		
12	R088Q03	0.681	0.112*		0.131*		
13	R088Q04T	0.647	0.107*		0.221*		
14	R088Q05T	0.639			0.288*		
15	R088Q07	0.648			0.195*		
16	R110Q01	0.645				0.187*	
17	R110Q04	0.737				0.276	
18	R110Q05	0.765				0.263*	
19	R110Q06	0.568				0.296	
20	R216Q01	0.647					0.282
21	R216Q02	0.803					
22	R216Q03T	0.829					0.106*
23	R216Q04	0.750					0.319
24	R216Q06	0.600					0.486
25	R236Q01	0.691				0.139*	
26	R236Q02	0.639					0.100*
ES			0.102	0.058	0.050	0.062	0.076

Note. G1= Reading literacy; A1-5 = 5 different articles; ES: effect size; underscored are specified loadings; only significant loadings at general factors and loading absolute value above 0.1 at special factors are presented.

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Summary

- Multiple general factors and special factors with different constraints on factorial correlation and residual
- Loading matrix, local dependence, mixed types of variables, missingness
- Regularization of loading structure
- Partially confirmatory structure

Recommendations for Practitioners

- effect size < 0.05 can be negligible
- small effect size (~.1) is ok, GPCFA will achieve good model estimation
- large effect size (~.2) might lead to overestimating for some parameters

Limitation

- Time-consuming
- Raw data are required

Further Plan

- Compare the performance of GPCFA with other generalized models
- Explore more large-scale empirical evidence